# Amortizing Caps

# Amortizing Cap Introduction

- A cap is a financial contract between two parties that provides an interest rate ceiling or cap on the floating rate payments.
- A cap actually consists of a series of European call options (caplets) on interest rates.
- An amortizing cap is a cap whose notional principal amount declines during the life of the contract.
- An accreting cap is a cap whose notional principal amount increases during the life of the contract.

# Amortizing Cap Introduction (Cont.)

- A floor is a financial contract between two parties that provides an interest rate “floor” on the floating rate payments.
- A floor consists of a series of European put options (floorlets) on interest rates.
- An amortizing floor is a floor whose notional principal amount declines during the life of the contract.
- An accreting floor is a floor whose notional principal amount increases during the life of the contract.

# Amortizing Cap Introduction (Cont.)

- Amortizing caps or floors are primarily used to hedge loans whose principal declines on a scheduled basis.
- Accreting caps or floors are primarily used to hedge construction loans whose principal increases on a scheduled basis to meet the expanding working capital requirements.
- Amortizing caps are frequently purchased by issuers of floating rate debt where the loan principal declines during the life.
- Accreting caps are frequently purchased by issuers of floating rate debt where the loan principal increases during the life.

# Amortizing Cap Introduction (Cont.)

- Amortizing floors are frequently purchased by purchasers of floating rate debt where the loan principal declines during the life.
- Amortizing floors are frequently purchased by purchasers of floating rate debt where the loan principal increases during the life.
- The amortizing/accreting cap holders wish to protect themselves from the increased financing costs that would result from an increase in interest rates.
- The amortizing/accreting floor holders wish to protect themselves from the loss of income that would result from a decrease in interest rates.

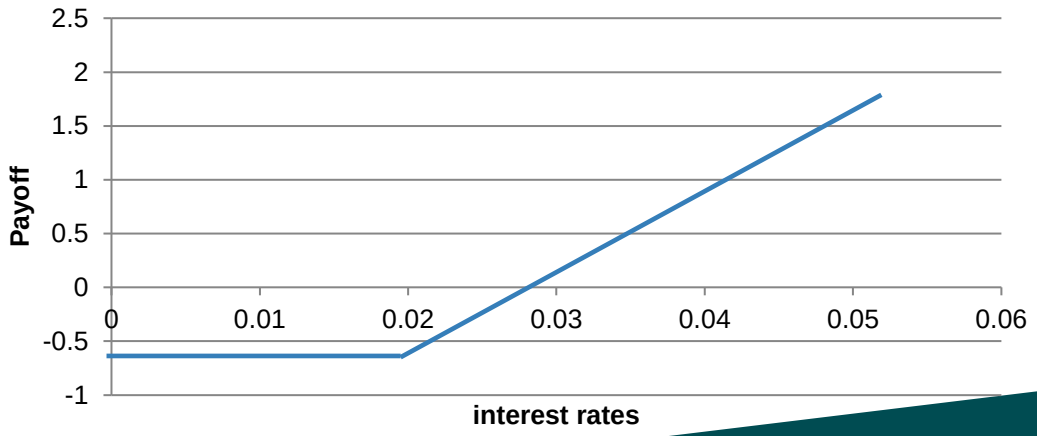
# Caplet Payoff

- The payoff of a caplet

$$Payoff = N * \tau * max(R - K, 0)$$

where N – notional; R – realized interest rate; K – strike; $\tau$ – day count fraction.

- Payoff diagram



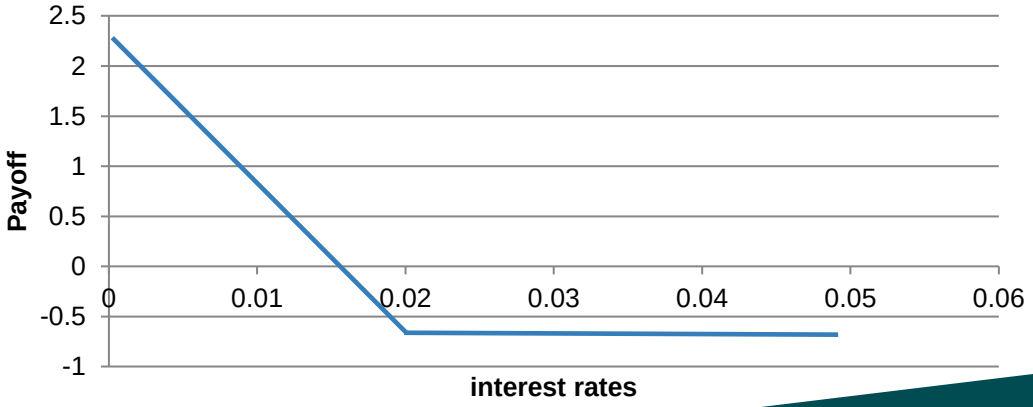
# Floorlet Payoff

- The payoff of a floorlet is given by

$$Payoff = N * \tau * max(K - R, 0)$$

where N – notional; R – realized interest rate; K – strike; $\tau$ – day count fraction.

- Payoff diagram



# Amortizing Cap

## Valuation

- The analytics is similar to a vanilla cap and floor except the principal amount used by each period may be different.
- The present value of a cap is given by

$$PV(0) = \sum_{i=1}^{n} N_i \tau_i D_i (F_i \Phi(d_1) - K\Phi(d_2))$$

where

$D_i = D(0, T_i)$ – the discount factor;

$F_i = F(t; T_{i-1}, T_i) = \left(\frac{D_{i-1}}{D_i} - 1\right) / \tau_i$ – the forward rate for period $(T_{i-1}, T_i)$.

$\Phi$ – the accumulative normal distribution function

$$d_{1,2} = \frac{\ln(\frac{F_i}{K}) \pm 0.5\sigma_i^2 T_i}{\sigma_i \sqrt{T_i}}$$

# Valuation (Cont.)

- The present value of a floor is given by

$$PV(0) = \sum_{i=1}^{n} N_i \tau_i D_i \big(K\Phi(-d_2) - F_i \Phi(-d_1)\big)$$

where

$D_i = D(0, T_i)$ – the discount factor;

$F_i = F(t; T_{i-1}, T_i) = \left(\frac{D_{i-1}}{D_i} - 1\right) / \tau_i$ – the forward rate for period $(T_{i-1}, T_i)$.

$\Phi$ – the accumulative normal distribution function

$$d_{1,2} = \frac{\ln(\frac{F_i}{K}) \pm 0.5\sigma_i^2 T_i}{\sigma_i \sqrt{T_i}}$$

# Amortizing Cap

## Notes

- Amortizing and accreting caps are valued via the Black model in the market.
- The forward rate is simply compounded.
- The first key to value a cap is to generate the cash flows. The cash flow generation is based on the start time, end time and payment frequency, plus calendar (holidays), business convention (e.g., modified following, following, etc.) and whether sticky month end.
- Then you need to construct interest zero rate curve by bootstrapping the most liquid interest rate instruments in the market. The most common used yield curve is continuously compounded.

# Amortizing Cap

## Notes (Cont.)

- Another key for accurately pricing an outstanding Cap is to construct an arbitrage-free volatility surface.
- The accrual period is calculated according to the start date and end date of a cash flow plus day count convention
- The formula above doesn't contain the last live reset cash flow whose reset date is less than valuation date but payment date is greater than valuation date. The reset value is

$$PV_{reset} = N_0 * \tau * max(R - K, 0) \text{ for cap}$$
$$PV_{reset} = N * \tau * max(K - R, 0) \text{ for floor}$$

which should be added into the above present value.

# Amortizing Cap

## Example

| Cap Terms and Conditions | | Notional Schedule | |
|---|---|---|---|
| Buy Sell | Sell | 9000000 | 2/6/2015 |
| Cap or Floor | Cap | 8785714.29 | 3/31/2015 |
| Strike | 0.025 | 8464285.72 | 6/30/2015 |
| Trade Date | 2/6/2015 | 8142857.15 | 9/30/2015 |
| Start Date | 2/6/2015 | 7821428.58 | 12/31/2015 |
| Maturity Date | 2/4/2019 | 7500000.01 | 3/31/2016 |
| Currency | USD | 7178571.44 | 6/30/2016 |
| Day Count | dcAct360 | 6857142.87 | 9/30/2016 |
| Rate type | Float | 6535714.3 | 12/30/2016 |
| Notional | 9000000 | 6214285.73 | 3/31/2017 |
| Pay Receive | Pay | 5892857.16 | 6/30/2017 |
| Payment Frequency | 1M | 5571428.59 | 9/29/2017 |
| Index Tenor | 1M | 5250000.02 | 12/29/2017 |
| Index Type | LIBOR | 4928571.45 | 3/30/2018 |

Reference:
https://finpricing.com/lib/EqRangeAccrual.html